Hierarchical dissolved oxygen control for activated sludge processes

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Abstract

A hierarchical controller for tracking the dissolved oxygen (DO) reference trajectory in activated sludge processes is proposed and investigated. The removal of nitrogen and phosphorous from wastewater is considered. Typically, an aeration system itself is a complicated hybrid nonlinear dynamical system with faster dynamics compared to the internal dynamics of the DO in a biological reactor. It is a common approach to neglect these dynamics and also important operational limitations of this system, such as the limits on the allowed frequency of switching of the blowers and on their capacity. The paper proposes a two-level controller to track the DO reference trajectory. The upper level control unit generates trajectories of the desired airflows to be delivered to aerobic zones of the biological reactor. A nonlinear model predictive control algorithm is applied to design this controller. The lower-level controller forces the aeration system to follow these set-point trajectories. The predictive control is also applied to design the lower-level controller based on piecewise linearised hybrid dynamics of the aeration system. The overall hierarchical controller is validated by simulation based on real data records.

Keywords: Hierarchical control; Predictive control; Optimising control; Hybrid systems; Wastewater systems

1. Introduction

An activated sludge wastewater treatment plant can be classified as a complex system due to its nonlinear dynamics, large uncertainty, multiple time scales in the internal process dynamics and multivariable structure. In addition, rather limited measurements are available during plant operation. Hence, the use of mathematical models for estimation of the plant state and prediction purposes is essential in the design and operation of the controller. However, there is a significant uncertainty in these models and their identification still remains an open problem. Hence, until recently, intensive work on the physical modelling of wastewater plants was rather separated from using these models for control design. Recent developments in control technologies and particularly in the model predictive control (MPC), handling an uncertainty, estimation, trajectory tracking in nonlinear systems and intelligent control triggered new research and applications in this field (e.g., Bechmann, Nielsen, Madsen, & Poulsen, 1998; Brdys & Zhang, 2001; Haarsma & Keesman, 1995; Keesman, Lukasse, & Van Straten, 1999; Kim, McAvoy, Anderson, & Hao, 2000; Lindberg & Carlsson, 1996a; Lukasse, Keesman, & Van Straten, 1998; Nielsen, 2001; Olsson & Newell, 1999; Puta, Reichl, & Franke, 1999; Sorensen, Thornberg, & Nielsen, 1995; Van der Veen, Babuška, & Verbruggen, 1999). A hierarchical multilayer control structure that utilises multiple time scales in the plant dynamics for robust optimised control of the biological wastewater treatment plants was proposed in Brdys, Grochowski, Gminski, Konarczak, and Drewa (2007). In this paper, the fast control sub-layer (FCS) is considered. A controller for an optimised tracking at the FCS of the dissolved oxygen (DO) concentration trajectory prescribed by the medium control sub-layer (MCS) is
synthesised. The DO control design was considered in e.g., Brdys and Konarczak (2001), Brdys and Diaz-Maiquez (2002), Chotkowski, Brdys, and Konarczak (2005), Haarmsma and Keesman (1995), Holmberg, Olsson, and Anderson (1989), Lindberg and Carlsson (1996a), Olsson and Newell (1999) and Yoo, Lee, and Lee (2002). Under a high variability of influent flow and of the concentration of pollutant, the plant operating point can vary considerably. Since the DO dynamics is highly nonlinear, a fixed parameter linear controller is not able to maintain a satisfactory tracking performance under the full range of operating conditions and requires on line parameter tuning (Yoo, Lee & Lee, 2002). A nonlinear, direct adaptive model reference controller was proposed by Chotkowski et al. (2005). However, there exist constraints on the magnitude and rate of change of the airflow delivered by the aeration system that can be properly handled only by a MPC algorithm. The MPC based on a linearised model of the DO dynamics was first considered by Haarmsma and Keesman (1995) for the removal of nitrogen. The MPC approach based on state space linear models identified in advance and fed by disturbances with modelled dynamics was investigated by Sanches and Katebi (2003) for the removal of nitrogen. The linear controller was favourably compared against a fixed parameter PI controller. The nonlinear predictive control was proposed by Brdys and Konarczak (2001) for the removal of nitrogen and phosphorus and further developed in Chotkowski et al. (2005). A superior performance of the nonlinear predictive controller over the linear one was demonstrated. A nonlinear fuzzy Takagi–Sugeno MPC based on multiple linear input–output models locally approximating the nonlinear DO dynamics was proposed by Brdys and Diaz-Maiquez (2002) and its improved computational efficiency while maintaining the nonlinear control performance was demonstrated. An output from all these controllers is a desired airflow trajectory to be provided by the aeration system, the controller actuator. This is shown for the aeration system at Kartuzy Wastewater Treatment Plant (WWTP), Northern Poland, with four aerobic biological reactor zones, illustrated in Fig. 1.

The aeration system shown in Fig. 1 is typical for WWTP with a continuous flow throughout the plant as opposed to the sequential batch reactor (SBR) WWTP and alternating activated sludge processes (Fikar, Chachuat, & Latifi, 2005). The control design methodology proposed in the paper applies to the continuous flow WWTP but is not directly applicable to the other two WWTP.

In this figure, \( S^\text{ref}_{ij} \) and \( S_{ij} \in \mathbb{T}_4 \) denote the prescribed and actual DO concentrations in the aerobic reactor zones, respectively, and \( Q_{\text{air},ij} \) are the airflows into these zones that control the DO concentrations.

Typically, an aeration system itself is a complicated hybrid nonlinear dynamical system with faster dynamics compared to the internal dynamics of the DO at a biological reactor. A common approach, including the work described above, is to neglect these dynamics and also the important operational limitations of this system imposed by the blower station. Once a blower has been switched off, it cannot be switched back on immediately, but only after some time period. If the time period is long compared to the rate of change of the airflow, then this...
needs to be taken into account by a controller scheduling the operation of the blowers. Otherwise, the desired airflow demand may not be met as the needed blower is disabled and cannot be activated when needed.

The energy cost due to blowing the air is taken into account at the MCS. However, due to the MCS time scale, only an average cost can be minimised when determining the optimised \( S_{o,ref} \) trajectories by this control layer (Brdys, Grochowski, Gminski, Konarczak, & Drewa, 2007). An exact evaluation of this cost is possible only at the FCS that directly considers the aeration system. The existing aeration system controllers are PLC-programmed PI algorithms that do not aim at all at combining the tracking functions with refining the cost optimisation. The paper further develops the results presented in Brdys, Chotkowski, Duzinkiewicz, Konarczak, and Piotrowski (2002), Chotkowski et al. (2005); Piotrowski, Duzinkiewicz, Konarczak, and Piotrowski (2002) , Piotrowski and Brdys (2004) and Piotrowski and Brdys (2005) and it is organised as follows: The structure of a hierarchical two-level controller for optimised DO concentration tracking is presented in Section 2. Section 3 presents a synthesis of the upper level controller (ULC). The aeration system modelling is described in Section 4. A hybrid model predictive controller at the lower control level is derived in Section 5. Application of the overall controller to Kartuzy WWTP is described in Section 6 and the simulation results based on real data records are presented. Section 7 concludes the paper.

2. Controller structure

The optimised DO concentration tracking controller is illustrated in Fig. 2. It operates within a multilayer hierarchical control structure (Brdys et al., 2007) at the FCS. This control system is directly coupled to the MCS where the optimised DO trajectories \( S_{o,ref} \) are generated for all the aerobic reactor zones. The objectives of the controller are to force the DO concentrations \( S_o \) in the zones to follow the prescribed references and at the same time to minimise the associated electrical energy cost due to blowing the air that is distributed into the zones throughout the aeration system pipes, throttling valves and diffusers. The control handles are located at the aeration system and they are: blower structure on/off, blower speeds and the angular positions of the throttling valves. These handles uniquely determine the trajectories of the airflows \( Q_{air,j} \) into the reactor zones (see Fig. 1). Hence, regarding the process variables, the resulting \( S_o \) mainly depends on the \( Q_{air} \), the external inflow \( Q_{in} \) into the reactor and its pollutant concentrations (disturbances). There are several external factors indirectly influencing the \( S_o \). For example, the wastewater temperature directly affects the \( Q_{air} \) limit \( S_{o,ref} \), hence it indirectly affects \( S_o \). The airflows \( Q_{air,j} \) can therefore be taken as the manipulated variables that couple the aeration system to the biological reactor. A multilevel decomposition of the control problem can now be applied (Brdys & Tatjewski, 2005) to structure the controller into a two-level hierarchy (see Fig. 2). The ULC uses the manipulated variables \( Q_{air} \) as its control outputs forcing \( S_o \) to follow \( S_{o,ref} \). The LLC acts as an actuating system and takes the ULC outputs that are the reference trajectories \( Q_{air,ref} \) of the airflows to be provided. The LLC uses the aeration system control handles in order to produce the airflow trajectories \( Q_{air} \) that follow the \( Q_{air,ref} \) prescribed by the ULC and to minimise the electrical energy cost due to blowing the air. The blower switching constraints are catered for by the LLC. The magnitude and rate limits on \( Q_{air} \) are naturally obeyed by the LLC and they are also taken into account at the upper control level so that feasible trajectories of manipulated variables can be prescribed by ULC to be delivered by the LLC.

Let us notice that the LLC operates on the fast time scale while the ULC operated on the slow one. The decomposition of the plant dynamics into two time scales described above does not achieve the exact separation of the two time scales, in contrast to the presented in Kumar and Daoutidis (2002). However, the overall plant dynamics considered in Kumar and Daoutidis (2002) are continuously differentiable. Hence, a standard singular perturbation method can be applied. Due to the on/off blower control variables, the aeration system dynamics is hybrid. To the best of the authors’ knowledge, a method for an exact time scale separation in hybrid systems is not available.

3. Upper level controller

3.1. An approach to controller design

The aerobic part of the biological reactor at Kartuzy WWTP is composed of four aerobic zones in cascade. Let us consider a general case with \( J \) such zones. Considering
the DO control problem, the aerobic zone $j$ can be separated from other parts of the WWTP with the inputs and outputs depicted in Fig. 3, where $Q_{in,j}$ [$m^3/h$], $Q_{out,j}$ [$m^3/h$], $Q_{air,j}$ [$m^3/h$], $S_{ain,j}$ [mgO$_2$/m$^3$], $S_{o,j}$ [mgO$_2$/m$^3$], $R_{r,j}$ [g/m$^3$h] are the waste inflow into the zone, waste outflow from the zone, airflow, influent DO concentration, DO concentration in the zone and the respiration, respectively.

The DO dynamics can be described as (Olsson & Newell, 1999):

$$
\frac{dS_{o,j}(t)}{dt} = \frac{Q_{a}(t)S_{ain,j}(t) - Q_{out,j}(t)S_{o,j}(t)}{V(t)} + k_L a(Q_{air,j}(t))(S_{o,sat} - S_{o,j}(t)) - \frac{S_{o,j}(t)}{K_o + S_{o,j}(t)}R_{r,j}(t), \tag{1}
$$

where $V$ [m$^3$], $K_o$ [g/m$^3$], $S_{o,sat}$ [mgO$_2$/m$^3$] are the zone volume, Monod constant and the DO concentration saturation limit, respectively.

The function $k_L a(Q_{air})$ in (1) describes the oxygen transfer and it is in general nonlinear and depends on the aeration actuating system and sludge conditions (Olsson & Newell, 1999). The third term in (1) denotes the respiration rate in the zone. Although the respiration rate may change rapidly, responding to fast changes of the influent or returned sludge flow the quantity, $R_{r,j}(t)$ changes much more slowly than $S_{o,j}(t)$ (Chotkowski et al., 2005). Typically, the first term in (1) is small and can be neglected to give

$$
\frac{dS_{o,j}(t)}{dt} = k_L a(Q_{air,j}(t))(S_{o,sat} - S_{o,j}(t)) - \frac{S_{o,j}(t)}{K_o + S_{o,j}(t)}R_{r,j}(t). \tag{2}
$$

There are another 18 nonlinear differential equations in the ASM2d model (Henze et al., 1999) needed to determine $R_{r,j}(t)$, requiring knowledge of the control input $Q_{air,j}(t)$, inflow $Q_{in,j}(t)$ and sophisticated information concerning the parameters of the wastewater inflow into the zone. This means that the state-space model of the DO concentration is described by a nonlinear dynamics of a very high order. Moreover, with the phosphorus reactions taken into account, the ASM2d model requires more than 60 parameters to be calibrated. Most of these parameters cannot be identified. Hence, the control problem is under a heavy uncertainty and adaptive or robust control technology is needed in order to handle it. Regardless of the obvious difficulties in developing control algorithm with good dynamic performance, the dimension of the resulting controller would be not tractable for an efficient implementation as the $S_{o,j}(t)$ dynamics is fast. However, let us notice that, as follows from (2), the uncertainty has an impact on $S_{o,j}(t)$ only through $R_{r,j}(t)$. Clearly, $S_{o,j}(t)$ influences $R_{r,j}(t)$ This cause-effect loop can be broken by considering $R_{r,j}(t)$ as an external disturbance signal. Indeed, assuming $R_{r,j}(t)$ is the disturbance input the overall dynamic model reduces to (2) and becomes the SISO nonlinear model. The control design problem becomes then vastly simplified. In the paper, this disturbance is estimated based on (2) and the measurements of $S_o$. Alternatively, it could be directly measured by using a respirometer.

3.2. Nonlinear model predictive controller (NMPC)

A model predictive technology (Maciejowski, 2001) is very suitable for the considered control problem as the disturbance $R_{r,j}(t)$ varies much more slowly than $S_{o,j}(t)$ (Chotkowski et al., 2005) and the actuator constraints can be effectively and accurately handled. A sufficiently accurate prediction of the respiration is needed on which to base the NMPC design that uses (2) for DO prediction purposes. Lindberg & Carlsson (1996b) have developed the Kalman filter for estimation of the respiration rate (the third term in (1)) together with parameters of the exponentially nonlinear function $k_L a(Q_{air})$. The estimates were used to feed their nonlinear controller the design of which was based on a feedback linearisation (Lindberg & Carlsson, 1996b). A simpler but still adequate approach is adopted in the present paper. Let us denote by $k_u$, $T_u$, $H^u_p$ the discrete time instant, DO sampling interval and prediction horizon at the upper control level, respectively. As the disturbance $R_{r,j}(t)$ varies much more slowly than $S_{o,j}(t)$, then given a good estimate of the instantaneous value of the disturbance, a sufficiently accurate disturbance prediction required by the predictive controller can be produced simply by extending this estimate over the prediction horizon. At the time instant $(k_u + 1)T_u$, the estimate $\hat{R}_{r,j}(k_u)$ of $R_{r,j}(k_uT_u)$ can be obtained by discretising Eq. (2), solving the resulting discrete time equation with respect to the unknown respiration value $R_{r,j}(k_u) \triangleq R_{r,j}(k_uT_u)$ and replacing the values $S_{o,j}(k_u)$, $S_{o,j}(k_u + 1)$ by the measurements $S_{o,j}^m(k_u)$, $S_{o,j}^m(k_u + 1)$ where $S_{o,j}(k_u) \triangleq S_{o,j}(k_uT_u)$. Hence,  

$$
\hat{R}_{r,j}(k_u) = - \frac{S_{o,j}^m(k_u + 1) - S_{o,j}^m(k_u)}{T_u} K_o + S_{o,j}^m(k_u) - k_L a(Q_{air,j}(k_u))(S_{o,sat} - S_{o,j}^m(k_u)) - \frac{S_{o,j}^m(k_u)}{K_o + S_{o,j}^m(k_u)}, \tag{3}
$$

where $Q_{air,j}(t) \triangleq Q_{air,j}(k_u)$, $t \in (k_uT_u, (k_u + 1)T_u)$.

There are several models of the function $k_L a(\cdot)$ (Olsson & Newell, 1999). These models involve constant parameters. The model structure, once identified by using one of the well-known model structure identification
techniques, remains fairly constant while the parameters and Monod constant $K_0$ need to be periodically identified by using one of the model calibration techniques.

As $\hat{R}_c(t)$ changes slowly, this value is taken as the estimate $R_c,\hat{J}(k_u + 1|k_u + 1)$ of $R_c,\hat{J}(k_u + 1)$ and then also as the prediction $R_c,\hat{J}(k_u + 1|k_u + 1)$ over the horizon $k_u + 1: k_u + 1 + H_p$ in the model-based optimisation problem to be solved by the NMPC at the instant $(k_u + 1)T_u$ (at the $k_u + 1$ control step). Let us introduce the aeration zone index set $J_u$, where $J_u = \{1, \ldots, J\}$.

It should be pointed out that a standard approach to handle the additive and unmeasurable disturbances in the linear MPC by introducing an integral action (Maciejowski, 2001) is not applicable here. This is because the disturbance is not additive and the dynamics of $S_o$ are nonlinear.

The NMPC performance function for zone $j \in J_u$ is defined as

$$J^U_j[Q_{air,j}(k_u|k_u), \ldots, Q_{air,j}(k_u + H_p - 1|k_u)]$$

$$= \sum_{i=1}^{H_p} \left( S_o,\hat{J}(k_u + i|k_u) - S_o^{\text{ref}}(k_u + i|k_u) \right)^2$$

$$+ \frac{\gamma_j}{2} \sum_{i=1}^{H_p-1} \left( Q_{air,j}(k_u + i|k_u) - Q_{air,j}(k_u + i - 1|k_u) \right)^2$$

$$+ \gamma_j \left( Q_{air,j}(k_u + 1|k_u) - Q_{air,j}(k_u|k_u) \right)^2$$

$$+ \sum_{i=1}^{H_p} \delta_j(Q_{air,j}(k_u + i - 1|k_u))^2, \quad (4)$$

where $Q_{air,j}(t) = Q_{air,j}(k_u + i|k_u)$, $t \in (k_u + i - 1)T_u$, $(k_u + i)T_u$ and $S_o,\hat{J}(k_u + i|k_u)$ is the model based prediction of $S_o,j(k_u + i)|k_u$ performed at the instant $k_u$.

The first term in (4) represents the tracking error. The second and third terms describe rates of changes of the control input over $H_p$ while the fourth term represents the control cost. The weights $\gamma_j, \delta_j, j \in J_u$ are tuning knobs used to achieve a desired compromise between the tracking error, the intensity of switching the blowers and the cost of the energy used due to pumping the air. An overall NMPC performance function reads:

$$J^U[Q_{air,1}(k_u|k_u), \ldots, Q_{air,J}(k_u + H_p - 1|k_u), \ldots, Q_{air,1}(k_u + H_p - 1|k_u)]$$

$$= \sum_{j \in J_u} J^U_j[Q_{air,j}(k_u|k_u), \ldots, Q_{air,j}(k_u + H_p - 1|k_u)]. \quad (5)$$

The instantaneous control magnitude and rate constraints are as follows:

$$Q_{air}^{\text{min}} \leq \sum_{j \in J_u} Q_{air,j}(k_u + i - 1|k_u) \leq Q_{air}^{\text{max}}, \quad i = \hat{J}; H_p \quad (6)$$

$$|Q_{air,j}(k_u + i - 1|k_u) - Q_{air,j}(k_u + i - 2|k_u)| \leq \Delta Q_{air,j}^{\text{max}}, \quad j \in J_u, i = \hat{J}; H_p. \quad (7)$$

At the time instants $k_uT_u$, the NMPC at the upper control level solves its optimisation task by minimising the performance function (4) with respect to the airflows subject to the constraints (6) and (7). The DO concentrations $S_o,j(k_u + i|k_u)$, $i = \hat{J}; H_p$ at the aerobic zones predicted over $H_p$ are calculated by using the discretised models (2). The respirations $R_c,\hat{J}(k_u + i - 1)$ in these models are replaced by their predictions $R_c,\hat{J}(k_u + i - 1|k_u)$ that are calculated according to (3) based on the DO measurements at the zones. The initial conditions $S_o,j(k_u|k_u)$ are taken from the measurements. This results in the optimised $\{Q_{air,j}(k_u|k_u), \ldots, Q_{air,j}(k_u + H_p - 1|k_u)\}$, $j \in J_u$, DO trajectories over the prediction horizon. Depending on the sampling interval and prediction horizon at the lower control level, certain values of the optimised airflows will be taken as the references for the LLC. The above procedure repeats at the time instant $(k_u + 1)T_u$ based on the updated respiration prediction and the initial condition taken from the DO measurements at $(k_u + 1)T_u$.

The ULC determines the control actions in a centralised manner. Its suboptimal decentralised version was presented by Brdyś et al. (2002).

4. Modelling the aeration system

Aeration is an important and expensive activity. The oxygen delivered into the aerobic zones by the aeration system is a fundamental component of the biological processes. It is essential to maintain the DO in a biological reactor at the right level. At least 0.2 mg dm$^{-3}$ is required for the nitrification processes to be carried out. An excessive DO concentration does not accelerate the nitrification but only increases the energy cost due to airflow pumping. The blower station contains several blowers that force air to the collector and then the airflow is divided among diffusers of the aeration tanks (see Fig. 1). This airflow must obey the minimum and maximum pressure constraints.

Two types of motors driving the blowers are used: fixed-speed motors and variable-speed motors. Variable-speed blowers are powered through inverters. Both types of the blowers have important operational limitations. A blower once switched off cannot be immediately switched back on, but only after a certain time period. The required airflow of each diffuser is maintained through positioning of a throttling valve. Generally, the relationship between airflow through the diffuser and the pressure drop across the diffuser is nonlinear. The diffusers are located inside the aerobic tank at a level of 4–5 m. The level implies a hydrostatic pressure in the tank. The generic diffusers are: disc diffuser, pipe diffuser and membrane diffuser and they offer different operational efficiency. Once the blower control actions have been applied, the controllers (typically proportional marked $R$ in Fig. 1) are used to provide the required airflows. Due to the throttle valve features (dead zone, saturation), it is in fact a nonlinear system. The airflows should not be throttled too much. This means that
a pressure at the collector node $p_c$, should have a suitable value to allow the throttling valves to work at as open a position as possible.

An aeration system itself is a complicated nonlinear dynamical system with fast dynamics. The blower switching constraints introduce additional (hidden) discrete time and with discrete valued variable type of dynamics. The control inputs at the blower station are integer valued in order to define which blowers are to be on/off and continuously valued in order to define the motor speeds. Hence, the aeration system as a whole is described by a nonlinear hybrid dynamics and the design of the LLC is not a trivial task. The control systems currently used in the wastewater industry are far from satisfactory. The cost of energy used is high and the control performance low. Again, the MPC technology will be applied to design the LLC. In this section, the model of the aeration system will be developed.

4.1. Physical modelling

The physical model of the aeration system consists of the following main element models: blower station, collector pipe and aeration segment unit. First the element models are presented following Piotrowski and Brdys (2005). Next an overall model is obtained by integrating the element models according to the aeration system structure as in Piotrowski and Brdys (2005).

4.1.1. Blower station

The blower station compresses air assuring appropriate pressure $p_c$ at the collector node. The station consists of blowers with fixed-speed and variable-speed motors. The fixed-speed blower can run with one speed or several constant speeds e.g., 1500 and 2900 r.p.m. for the two speeds. In contrast to the fixed-speed blowers, the variable-speed blower can run with speed varying continuously over a certain range. The blowers are in parallel, and the binary vector $x_b$ defines which of the blowers are on and off. Thus, $x_b$ defines the blower station operating structure, hence its state.

4.1.2. Collector pipe

The collector pipe is treated as the fluid flow capacitance with negligible fluid-flow resistance (Woods & Lawrence, 1997) and its electrical analogy is illustrated in Fig. 4.

Applying the standard mass balance principle at the collector node yields:

$$\frac{dp_c}{dt} = \frac{1}{C_c} (Q_b - Q_c) \text{ and } C_c = \frac{V_c}{p_c}$$

where $C_c$, $Q_c$, $V_c$ and $p_c$ are the collector fluid-flow capacitance, collector flow, collector volume and pressure at the collector node, respectively.

The above differential equation is nonlinear. As the fluid flow resistance of the collector is negligible, $p_c = p_b$.

4.1.3. Aeration segment unit

The aeration segment unit system is composed of the throttling valve, diffuser and collector-diffuser pipe. As the fluid-flow resistance of the collector-diffuser pipe is negligible, the unit system is modelled as a fluid-flow

The blower station model is obtained in a straightforward manner by utilising the blower models (8) and the parallel station structure:

$$Q_b = f_b(x_b, \Delta p_b, n_b) \triangleq \sum_{i \in I_b} f_{b,i}(x_{b,i}, \Delta p_{b,i}, n_{b,i}),$$

where $Q_b$, $p_b$, $n_b$ are the airflow into the blower node, pressure at the blower station node and atmospheric pressure, respectively;

$$\Delta p_b = p_b - p_a = \Delta p_{b,i}; \quad i \in I_e \cup I_f,$n_b = [n_{b,1}, \ldots, n_{b,i}, \ldots, n_{b,I_e+I_f}].$$

The binary vector $x_b$ defines which of the blowers are on and off. Thus, $x_b$ defines the blower station operating structure, hence its state.
capacitance $C_{d,j}$ catering for the collector–diffuser pipe with two resistances $R_{v,j}$, $R_{d,j}$ regarding the throttling valve and diffuser, respectively. This is followed by a hydrostatic pressure source (hydrostatic pressure of sewer) in cascade. The system and its electrical analogy are illustrated in Fig. 5.

The throttling valve resistance is a nonlinear function of the angular position of the valve. There are limits on the valve opening angle and on the pressure of airflow entering the main pipe. The limits are modelled by using the initial opening angle, dead zone and saturation. The manufacturer’s data have been used to determine an expression for the valve resistance as:

$$ R_{v,j} = k \left( \frac{a_4}{\phi_j^{b_4}} \right)^{c_4}; \quad j \in J_u, \quad (13) $$

where $R_{v,j}$, $\phi_j$ are the throttling valve resistance and angular valve position, respectively; $k = 9$, $a_4 = 120000$, $b_4 = 3$, $c_4 = 1$.

The flow against pressure drop relationship for the valve reads:

$$ \Delta p_{v,j} = R_{v,j}(\phi_j)Q_j^p; \quad j \in J_u, \quad (14) $$

where $Q_j$ is the airflow through the valve.

In a compact form:

$$ \Delta p_{v,j} = f_{v,j}(Q_j, \phi_j); \quad j \in J_u. \quad (15) $$

An example of comparison of the valve model with the valve catalogue data is illustrated in Fig. 6, showing a reasonable modelling accuracy.

In Fig. 5, the $R_{d,j}, C_{d,j}$ circuit together with the voltage source $\Delta p_{h,j}$ constitutes an electrical representation of the diffuser and collector–diffuser pipe system. The resistance $R_{d,j}$ represents the diffuser. It is assumed that pipe diameter is constant along the pipe and also, that it is large enough so that the pipe resistance can be neglected. Hence, the pipe is represented only by the fluid flow capacitance $C_{d,j}$.

Let us now consider the diffuser resistance. For steady-state conditions, the diffuser is described by a nonlinear function $Q_{air,j} = f_{d,j}(\Delta p_{d,j})$ relating the airflow through the diffuser and the pressure drop across the diffuser. The manufacturer’s data have been used to determine a piecewise approximation of this function as:

$$ Q_{air,j} = \begin{cases} \frac{\Delta p_{d,j} - \Delta p_{d,j}^{open}}{R_{d,j}} & \text{for } \Delta p_{d,j} \geq \Delta p_{d,j}^{open} \quad j \in J_u, \\ 0 & \text{otherwise} \end{cases} \quad (16) $$

where $Q_{air,j}$, $R_{d,j}$, $\Delta p_{d,j}$ are the airflow into aeration zone $j$, fluid-flow diffuser resistance and pressure drop across diffuser, respectively.

An example of comparison of the function $f_{d,j}(\cdot)$ and its piecewise approximation is shown in Fig. 7. The dead zone
\[ \Delta p_{d,j}^{\text{open}} \] and the diffuser response when open can be clearly seen and \( \Delta p_{d,j}^{\text{open}} = 2.25 \text{kPa} \).

The model completes with two parameters describing the diffuser dip depth and a number of diffusers in aeration tank. Without loss on generality it is assumed that all the diffusers are described by the same functions. It will be further assumed that all diffusers are open and this is guaranteed by \( p_c \geq p_{c_{\text{min}}} \). The above model was validated on real data records.

Finally, the dynamics of the diffuser airflow can be described as (see Fig. 5):

\[ R_{d,j} C_{d,j} \frac{dQ_{\text{air},j}}{dt} + Q_{\text{air},j} = Q_{j} \quad j \in J_a. \] (17)

In reality, the pipe diameter varies along the pipe. The above model allows modelling of the varying diameter pipe as well. The pipe is first decomposed into constant diameter sub-pipes and the model is applied to each of the sub-pipes yielding the corresponding \( R_{d,j} C_{d,j} \) circuits (see Fig. 5). The sub-models are then connected in cascade in order to produce final model of the diffuser–collector pipe system. The values of the electrical components in the final model must be identified from real data measurement records by applying system identification methods. As the diffuser outflow pressure is nearly constant, the \( C_{d,j} \) can also be treated as having a constant value.

The hydrostatic pressure in the aeration tank is modelled as:

\[ \Delta p_{h,j} = \rho g h_j; \quad j \in J_a, \] (18)

where \( \rho \), \( g \), and \( h_j \) are the wastewater density at aeration tank, acceleration due to gravity and the height of the diffuser in the wastewater at the aeration tank, respectively; since \( h_j \) is typically constant, \( \Delta p_{h,j} \) is constant as well.

The total airflow delivered to the aeration segment unit is

\[ Q_c = \sum_{j \in J_a} Q_j, \] (19)

The pressure drop across the aeration segment unit with a diffuser that is open is described by the equation (see Fig. 5):

\[ p_c - p_a = \Delta p_{c,j} + \Delta p_{d,j} + \Delta p_{h,j}; \quad j \in J_a. \] (20)

4.1.4. Overall physical model

Modelling the aeration system elements has now been completed and a whole model of the interconnected system can now be derived. Similarly to interconnected networks, not only the model structure will be required but also new physical relationships will emerge as the result of connecting the elements. Due to the modelling methodology applied, the overall model enjoys the feature that it is easy to redesign the element models and change the parameters. The electrical circuit in Fig. 8 is used to summarise the derived physical model of the aeration system.

In this model the control variables are binary variables \( x_b \) determining the blower on or off status, continuous variables \( n_{b,i}, i \in I_b \) determining speeds of the variable-speed blowers, discrete variables \( n_{h,i}, i \in I_f \) determining gears of the fixed-speed blowers and continuous variables \( Q_{j}, j \in J_a \) determining angular positions of the throttling valves.

The output variables are pressure at the collector node \( p_c \) and the airflows \( Q_{\text{air},j}, j \in J_a \).

The following assumptions are made: the blower station can be represented by a current source and only the capacitances of the main collector pipe and aeration segment units are taken into account as the dominant ones. The circuit in Fig. 8 will now be used for calculating the segment unit flows into the aeration tanks: \( Q_{\text{air},j}, j \in J_a \).
Following the circuit input–output electrical flow and applying the Eqs. (12), (19), (9), (20), (17) and (15) the overall model can be presented as follows:

\[
\frac{dp_c}{dt} = \frac{p_c}{V_c} \left[ \sum_{i \in I_f} f_{h,i} (x_{h,i}, p_c, n_b, i) - \sum_{j \in J_d} Q_j \right], \quad (21)
\]

\[
\frac{dQ_{air,j}}{dt} = \frac{1}{R_{d,j}C_{d,j}} (Q_{j} - Q_{air,j}); \quad j \in J_d, \quad (22)
\]

\[
p_c = \Delta p_{open} + R_{d,j} Q_{air,j} + f_{v,j} (Q_j, \varphi_j) + \Delta p_{h,j} + p_a; \quad j \in J_d, \quad (23)
\]

where \( n_b, x_b, \varphi_j, j \in J_d \) are control variables, \( p_c, Q_{air,j}, j \in J_d \) are state variables being at the same time the output variables.

The blower states \( x_b \) in this model are the input variables and their dynamics will be derived in the next section.

This model is described by set of nonlinear differential and algebraic equations where the algebraic Eq. (23) implicitly determine flows \( Q_j, j \in J_d \) in terms of the states and control inputs.

4.1.5. Selected simulation results for Kartuzy WWTP

The above model was verified based partly on data records from Kartuzy WWTP and also based on the data available from the documentation of the characteristics of the system elements. The control inputs over a 1-h period are shown in Figs. 9–11. The corresponding model outputs are illustrated in Figs. 12 and 13.

Apart from validating the model, the results show that the internal dynamics of the aeration system is fast when compared to the control input rates.

4.2. Modelling the dynamics of the blower switching constraints

The blower states can be changed immediately in a controlled manner every \( k_f T_f \) time units, where \( k_f \) is an integer variable representing discrete time at the lower control level while \( T_f \) is the sampling interval. The switching time is assumed zero so that with \( x_b(k_f) = x_b(k_f T_f) \), \( x_b(t) = x_b(k_f T_f + 1)T_f = x_b(k_f T_f + 1), \quad t \in (k_f T_f, (k_f + 1)T_f) \).

Every blower can be switched on or off implying changes of the blower state between 0 and 1. Let us define the switching control variables for the blower \( i \in I_h \) as the binary and discrete time variables \( u_{on}^{(i)}, u_{off}^{(i)} \). If \( u_{on}^{(i)}(k_f) = 1 \) and \( x_{h,i}(k_f) = 0 \), then the blower is switched on and
\( x_{b_h}(k+1) = 1 \). If \( u_{b_h}^{on}(k) = 0 \) and \( x_{b_h}(k) = 1 \), then the blower status remains unchanged and \( x_{b_h}(k+1) = 1 \). The impact of \( u_{b_h}^{off} \) on the blower switched off status is similar. The blower switching control variables are dependant as it is not viable to switch the blower on and off at the same time. Hence, the following constraint is introduced:

\[ u_{b_h}^{off}(k) + u_{b_h}^{on}(k) \leq 1; \quad i \in I_f \cup I_v. \]  

(24)

Moreover, in order to maintain the range \([0,1]\) for the blower state trajectory \( x_{b_h}(k) \) in the equation describing its dynamics, the pairs \((x_{b_h}(k), u_{b_h}^{off}(k)) = (0,1)\) and \((x_{b_h}(k), u_{b_h}^{on}(k)) = (1,1)\) are excluded by introducing the following constraints:

\[ x_{b_h}(k) - u_{b_h}^{off}(k) \geq 0, \]
\[ x_{b_h}(k) + u_{b_h}^{on}(k) \leq 1 \quad i \in I_f \cup I_v. \]  

(25)

Indeed, the constraints (25) prevent commanding the blower already switched off to be switched off again and also the blower already switched on to be switched on again. An overall blower station state dynamics can now be described for \( i \in I_b \) as:

\[ x_{b_h}(k+1) = x_{b_h}(k) - u_{b_h}^{off}(k) + u_{b_h}^{on}(k). \]  

(26)

As it has been stated before, a blower switched off cannot be switched on immediately but only after certain time period. Let us denote by \( N_s T_l \) the forced blower standstill period. In order to derive dynamical constraints on the values of the switching control variables \( u_{b_h}^{off}, u_{b_h}^{on} \) that meet this condition, a dedicated state variable \( S_{b_h}(k) \) is introduced to denote a number of steps over which the blower \( i \) has remained switched off till the time instant \( k_t \). A recursive equation driving evolution of \( S_{b_h} \) over time can be written as

\[ S_{b_h}(k+1) = (1 - x_{b_h}(k+1)) S_{b_h}(k) + (1 - x_{b_h}(k+1)). \]

(27)

Substituting (26) into the above equation gives

\[ S_{b_h}(k+1) = (1 - x_{b_h}(k)) S_{b_h}(k) + (u_{b_h}^{off}(k) - u_{b_h}^{on}(k)) S_{b_h}(k) + 1 - x_{b_h}(k) + u_{b_h}^{off}(k) - u_{b_h}^{on}(k). \]  

(28)

The standstill switching conditions for the station blowers can now be expressed as follows:

\[ S_{b_h}(k+1) - N_s T_l \geq 0; \quad i \in I_b. \]  

(29)

Eqs. (26) and (27) together with the constraints (24), (25) and (28) describe the blower station switching constraint dynamics that cater for the limit on the blower switching frequency. This, combined with the Eqs. (21), (22) and (23), constitute a complete model of the aeration system. Notice that the model is hybrid and nonlinear.

5. Lower-level controller

As described earlier, the control problem at the lower level is to schedule the blowers and determine their speeds,
determine the throttling valve openings so that the airflow demand prescribed by the ULC is met with the lowest possible energy cost and the constraints are satisfied. Hence, the control variables at the lower control level are: \( u_{\text{off}}^{pc}, u_{\text{on}}^{pc}, n_{b}, \phi \) composed of \( u_{\text{off}}^{pc}, u_{\text{on}}^{pc}, n_{b}, i \) in \( I_{b} \cup J_{a} \) and \( \phi, \) \( j \) in \( J_{a} \), while \( Q_{\text{air}}, j \) \( j \) in \( J_{a} \) are relevant for the ULC output variables and \( p_{c} \) is a relevant output for the LLC performance. The switching control inputs are implemented in the system by simple PLCs. The speed control inputs are forced in the system by inverters. The throttling valve openings are forced by local PLC loops around the valves (see Fig. 1) with the corresponding output airflows as the set-points and the openings as the loop inputs. Due to the nonlinearity of the input–output model and constraints and because the variables involved are mixed integers, a hybrid nonlinear model predictive control (HN MPC) will be applied to derive the LLC algorithm. First, the constraints and performance function of the HN MPC will be formulated. This will lead to a nonlinear mixed integer formulation of the HN MPC optimisation task. There is no solver available at present that can solve such a problem in a robust manner, meeting the time constraints set by the on-line implementation. Therefore, the optimisation problem will be linearized by introducing additional continuous and discrete variables to obtain mixed integer linear (MIL) formulation that can, for example, be effectively approached by the CPLEX solver.

5.1. Constraints

There are two types of constraints to be accounted for: the constraints implied by the physical aeration system model and the switching constraints.

5.1.1. Model-based constraints

The speed and valve opening control variable trajectories are piecewise constant and determined by sequences of the vectors \( n(ki), \phi(ki) \) \( ki \) such that

\[ n_b(i) = n_b(ki), \quad t \in (k_i T_a, (k_i + 1)T_a] \quad \text{and} \quad \phi(t) = \phi(k_i), \quad t \in (k_i T_a, (k_i + 1)T_a]. \]

The aeration system model response over \( t \in (k_i T_a, (k_i + 1)T_a] \) is then determined by \( n(ki), \phi(ki), x(k_i + 1) \) to produce the outputs \( p_c(t), Q_{\text{air}}(t), Q(t) \). As it has been seen in Section 4.1, the steady-state values of the outputs are very quickly achieved. Hence, \( p_c(t) = p_c((k_i + 1)T_a) \), \( Q_{\text{air}}(t) = Q_{\text{air}}((k_i + 1)T_a) \), \( Q(t) = Q((k_i + 1)T_a) \) and can be assumed over \( t \in (k_i T_a, (k_i + 1)T_a] \). Thus, (22) implies that \( Q((k_i + 1)T_a) = Q_{\text{air}}((k_i + 1)T_a), j \in J_a \).

Denoting \( p_c(k_i + 1) \equiv p_c((k_i + 1)T_a) \), \( Q_{\text{air}}(k_i + 1) \equiv Q_{\text{air}}((k_i + 1)T_a) \), Eqs. (21)–(23) yield the steady-state model of the aeration system in the form:

\[
\begin{align*}
\sum_{i \in T_a \cup J_a} f_{h,i}(x_{h},(k_i + 1), p_c((k_i + 1)), n_{b,i}(k_i)) & - \sum_{j \in J_a} Q_{\text{air},j}(k_i + 1) = 0, \\
p_c(k_i + 1) &= \Delta P_{\text{dyn}}^{open} + R_{d,j}Q_{\text{air},j}(k + 1) + f_{u,j}(Q_{\text{air},j}(k + 1), \phi(k_i)) + \Delta P_{b,j} + P_{air}, j \in J_a, \quad (30)
\end{align*}
\]

with the simple bound constraints on the variables:

\[
\begin{align*}
p_{c}^{\text{min}} \leq p_c(k_i + 1) & \leq p_{c}^{\text{max}}, \quad x_{b,i}(k_i + 1) \in [0, 1], \quad i \in I_b; \\
Q_{\text{air},j}(k_i + 1) & \geq 0, \quad \phi_{c}^{\text{min}} \leq \phi(k_i) \leq \phi_{c}^{\text{max}}, \quad j \in J_a; \\
n_{b,i}(k_i) & \in N_v, \quad i \in I_v; \quad n_{b,i}(k_i) \in N_j, \quad i \in I_f. \quad (31)
\end{align*}
\]

As the model is implicit the decision variables in the HN MPC using this model are: \( x_b(k_i + 1), n_b(k_i), p_c(k_i + 1), Q_{\text{air},j}(k_i + 1), \phi(k_i) \) The model Eqs. (29) and (30) and the inequalities (31) are the equality and inequality constraints, respectively, in the HN MPC optimisation problem.

5.1.2. Switching constraints

Eqs. (26) and (27) describing the blower switching dynamics constitute the equality constraints, while the inequalities (24), (25), and (28) form the inequality constraints in the HN MPC optimisation problem. Notice that the optimisation problem is dynamical and this is entirely due to the switching frequency limit. New decision variables introduced into the HN MPC by the constrained switching dynamics are: \( u^{\text{off}}, u^{\text{on}}, S_b \).

5.2. Performance function

The electrical energy cost due to blowing the air is proportional to the collector pressure \( p_c \). Hence, the performance function is formulated as:

\[
J^L = \sum_{i=1}^{n_f} \left[ \eta(k_i + i - 1) p_c((k_i + i)T_a) + \sum_{j \in J_a} Q_{\text{air},j}(k_i + i) - Q_{\text{air},j}(k_i + i) \right], \quad (32)
\]

where \( H^L_p \) denotes the prediction horizon at the lower level.

The second term in (32) represents the airflow tracking error. The absolute value was taken instead of a traditional square in order to exactly linearise the tracking term. The coefficient \( \eta \) includes the time varying electricity tariff. The reference airflows \( Q_{\text{air},j}^{\text{ref}}((k_i + i)T_a) \) are prescribed by the ULC.

5.3. Hybrid nonlinear model predictive controller (HN MPC)

At the time instants \( k_0 T_a \), the HN MPC at upper control level solves its optimisation task by minimising the performance function (32) with respect to the following
decision variables:
\[ x_b(k_i + i|k_i), \ u^\text{off}(k_i + i - 1|k_i), \ u^\text{on}(k_i + i - 1|k_i), \]
\[ S_b(k_i + i|k_i), n_i(k_i + i - 1|k_i), p_i(k_i + i|k_i), \]
\[ Q_{a_i}(k_i + i|k_i), \ \phi(k_i + i - 1|k_i), \ i = 1 : H_p^t, \]
which include both the control and output variables over the prediction horizon \( H_p^t \). Given the control inputs at \( k_i \) over \( H_p^t \), the outputs \( Q_{a_i}(k_i + i|k_i) \), \( p_i(k_i + i|k_i) \) and states \( x_b(k_i + i|k_i) \), \( S_b(k_i + i|k_i) \) over \( H_p^t \) are predicted by solving the mixed dynamic and implicit steady-state hybrid model Eqs. (26), (27) and (29), (30), respectively. The initial states \( x_b(k_i|k_i) \), \( S_b(k_i|k_i) \) needed to perform the predictions are taken from the measurements. This is an information feedback from the system that allows correcting of the model-based states when the switching hardware errors occur. These equations together with the inequalities (24), (25), (28) and (31) constitute the optimised trajectories over \( H_p^t \) of the switching variables \([u^\text{off}(k_i + i - 1|k_i), u^\text{on}(k_i + i - 1|k_i)]\), the blowers speeds \([n_i(k_i + i - 1|k_i)]\) and the throttling valve openings \([\phi(k_i + i - 1|k_i)]\).

Only the first control step values \( u^\text{off}(k_i|k_i), u^\text{on}(k_i|k_i), n_i(k_i|k_i), \phi(k_i|k_i) \) are applied to the aerodynamic system. At the time instant \((k_i + 1)T_f\), the above procedure repeats based on the updated airflow references \( Q_{a_i}(k_i + 1 + i|k_i + 1) \) and the switching state initial conditions \( x_b(k_i + 1|k_i + 1), S_b(k_i + 1|k_i + 1) \) taken from the measurement at \((k_i + 1)T_f\) and with the constraints:
\[-z_{i,j} \leq Q_{a_i}(k_i + i|k_i) - Q_{a_i}(k_i + i|k_i) \leq z_{i,j}, \]
\[z_{i,j} \geq 0, \ i = 1 : H_p^t, \ j \in J_a.\] (34)

5.4. Linearisation and hybrid linear model predictive controller

The performance function and constraints of the HN MPC optimisation problem are nonlinear and the variables involved are mixed integer. These will be now linearised so that an approximation of the solution to the problem can be obtained robustly. This is a common approach (Williams, 1994; Flouds, 1995) that has been successfully applied, for example, to water distribution systems by Brydys and Chen (1995), wastewater systems by Piotrowski and Brydys (2005) and industrial processes by Bemporad and Morari (1999).

5.4.1. Performance function

The second term in (32) is nonlinear. The new optimisation variables \( z_{i,j} \) and constraints are introduced to exactly linearise unconstrained optimisation of \( J^L \) by a linear constrained optimisation with the performance function \( J_{\text{lin}}^L \), where
\[ J_{\text{lin}}^L = \sum_{i=1}^{H_p^t} \left[ \eta(k_i + i - 1) p_i(k_i + i|k_i) T_f + \sum_{j \in J_a} z_{i,j} \right] \] (33)

5.4.2. Throttling valve model

The functions \( f_{c,j}(\cdot) \) in (30) are nonlinear (see (14) and (15)). They will be piecewise plane approximated by linear relationships with mixed integer variables. Given \( j \), let us set up a grid \( \{Q_{a_i}^m, \varphi_j^m\} \), where \( m = 1 : N_{Q_{a_i}}, n = 1 : N_{\varphi_j} \) over an operating range of the \( j \)th valve, where \( Q_{a_i}, \varphi_j \in [Q_{a_i}^\text{min}, Q_{a_i}^\text{max}], \varphi_j^\text{min}, \varphi_j^\text{max} \).

The pressure drops at the grid points are: \( \Delta p_{c,n}^m = f_{c,j}(Q_{a_i}^m, \varphi_j^m) \). The grid points partition the operating range into rectangular cells. Without loss of generality let us consider the cell defined by the points with \( m = 1, 2 \) and \( n = 1, 2 \). The points \( \{Q_{a_i}^1, \varphi_j^1\}, \{Q_{a_i}^2, \varphi_j^2\}, \{Q_{a_i}^N, \varphi_j^N\} \) uniquely determine a plane that approximates \( f_{c,j}(\cdot) \) over the left half of the triangular cell (see Fig. 14) and values of the linear approximation \( f_{c,j}^\text{lin}(\cdot) \) over this set can be calculated as:
\[ f_{c,j}^\text{lin}(Q_{a_i}, \varphi_j) = \lambda_j^1 Q_{a_i} + \lambda_j^2 \varphi_j + \lambda_j^3 Q_{a_i} \varphi_j + \lambda_j^4 Q_{a_i}^2 + \lambda_j^5 \varphi_j^2 + \lambda_j^6 Q_{a_i} \varphi_j^2 + \lambda_j^7 Q_{a_i}^2 \varphi_j + \lambda_j^8 Q_{a_i} \varphi_j^2 + \lambda_j^9 Q_{a_i}^2 \varphi_j^2, \] (35)

\[ Q_{a_i} \lambda_j^1 + \lambda_j^2 \varphi_j + \lambda_j^3 Q_{a_i} \varphi_j + \lambda_j^4 Q_{a_i}^2 + \lambda_j^5 \varphi_j^2 + \lambda_j^6 Q_{a_i} \varphi_j^2 + \lambda_j^7 Q_{a_i}^2 \varphi_j + \lambda_j^8 Q_{a_i} \varphi_j^2 + \lambda_j^9 Q_{a_i}^2 \varphi_j^2, \] where \( \lambda_j^1, \lambda_j^2, \lambda_j^3, \lambda_j^4, \lambda_j^5, \lambda_j^6, \lambda_j^7, \lambda_j^8, \lambda_j^9 \) are nonnegative real numbers and \( \lambda_j^1 + \lambda_j^2 + \lambda_j^3 = 1 \).

Fig. 14. Planewise linearisation of the throttling valve nonlinear model.
The new variables \( \tilde{\lambda}_{j}^{1,1}, \tilde{\lambda}_{j}^{2,1}, \tilde{\lambda}_{j}^{1,2} \) can be viewed as the weights associated with the grid nodes \( \{Q_{lin,j}^{0}, \varphi_{j}\} \), \( \{Q_{air,j}^{0}, \varphi_{j}\} \) and \( \{Q_{air,j}^{1}, \varphi_{j}\} \), respectively. They select points over the set and then weight the \( f_{v,k}(\cdot) \) values at these points to produce the approximations according to (35). The approximation over the right half of the triangular cell is obtained in a similar way but using the point \( \{Q_{air,j}^{0}, \varphi_{j}\} \) instead of \( \{Q_{air,j}^{1}, \varphi_{j}\} \). An overall two different plane approximation of a whole cell can be written as:

\[
\begin{align*}
\hat{f_{v,j}}^{lin}(Q_{air,j}, \varphi_{j}) &= \sum_{m=1}^{2} \sum_{n=1}^{2} \tilde{\lambda}_{j}^{m,n} \Delta\rho_{m,n}, \\
\text{where } Q_{air,j} &= \sum_{m=1}^{2} \sum_{n=1}^{2} \lambda_{j}^{m,n} Q_{air,j}^{m,n}, \quad \varphi_{j} = \sum_{m=1}^{2} \sum_{n=1}^{2} \lambda_{j}^{m,n} \varphi_{j}^{m,n} \\
\sum_{m=1}^{2} \sum_{n=1}^{2} \lambda_{j}^{m,n} &= 1, \quad \lambda_{j}^{m,n} \geq 0, \quad m = 1, 2, \quad n = 1, 2
\end{align*}
\]

under an additional condition guaranteeing that correct grid nodes are selected to produce the correct approximating planes, that is either \( \tilde{\lambda}_{j}^{1,1} \) or \( \tilde{\lambda}_{j}^{2,1} \) can be nonzero but not both or the set \( \{\tilde{\lambda}_{j}^{1,1}, \tilde{\lambda}_{j}^{2,2}\} \) is SOS2 (Wiliams, 1994):

\[
\begin{align*}
\tilde{\lambda}_{j}^{1,1} - \delta_{j}^{1,1} &\leq 0, \quad \tilde{\lambda}_{j}^{2,2} - \delta_{j}^{2,2} \leq 0, \quad \delta_{j}^{1,1} + \delta_{j}^{2,2} = 1,
\end{align*}
\]

where \( \delta_{j}^{1,1} \) and \( \delta_{j}^{2,2} \) are binary variables.

Finally, the relationships (36) and (37) will now be extended to produce a MIL variables approximation of the valve model over the whole operating range as follows:

\[
\begin{align*}
\hat{f_{v,j}}^{lin}(Q_{air,j}, \varphi_{j}) &= \sum_{m=1}^{N_{Q_{j}}} \sum_{n=1}^{N_{\varphi_{j}}} \lambda_{j}^{m,n} \Delta\rho_{m,n}, \\
Q_{air,j} &= \sum_{m=1}^{N_{Q_{j}}} \sum_{n=1}^{N_{\varphi_{j}}} \lambda_{j}^{m,n} Q_{air,j}^{m,n}, \quad \varphi_{j} = \sum_{m=1}^{N_{Q_{j}}} \sum_{n=1}^{N_{\varphi_{j}}} \lambda_{j}^{m,n} \varphi_{j}^{m,n}, \\
\sum_{m=1}^{N_{Q_{j}}} \sum_{n=1}^{N_{\varphi_{j}}} \lambda_{j}^{m,n} &= 1, \quad \lambda_{j}^{m,n} \geq 0, \quad m = 1: N_{Q_{j}}, \quad n = 1: N_{\varphi_{j}},
\end{align*}
\]

where for any \( n = \frac{1}{N_{\varphi_{j}}}, \) the ordered sets of variables \( \{\lambda_{j}^{m,n}\}_{m=1}^{N_{Q_{j}}} \) are SOS2 and for any \( m = \frac{1}{N_{Q_{j}}}, \) the ordered sets of variables \( \{\lambda_{j}^{m,n}\}_{n=1}^{N_{\varphi_{j}}} \) are SOS2.

Notice that the SOS2 conditions imply that at most two adjacent weights in the sets are nonzero. Hence, the corresponding grid nodes selected do indeed define a cell.

In summary, the planewise linearised valve model replaces \( f_{v,j}(\cdot) \) by \( \hat{f_{v,j}}^{lin}(\cdot) \) defined by (38) and (39) as the function of SOS2 ordered variables \( \lambda_{j}^{m,n} \) constrained by (40) and (41), where in the (41) new binary variables \( \delta^{m,n}_{j}, \) \( m = 1: N_{Q_{j}}, \quad n = 1: N_{\varphi_{j}} \) are introduced. Wherever else

### 5.4.3. Switching dynamics model

Inspecting the model of the switching dynamics shows that the nonlinear terms appear only in Eq. (27) as \( x_{b,h}(k) S_{h}(k) \) and both of them are in the form of a product of a binary variable and another variable. Applying the result from Bemporad and Morari (1999) to the first term, its exact mixed integer constrained linearisation can be derived as follows. Let us introduce the new variable \( S_{h,i}(k) \) that is defined by (8). Consider the first term and let us denote by \( S_{h,i}^{min} \) and \( S_{h,i}^{max} \) the upper and lower bounds on \( S_{h,i}(k) \), respectively. The exact linearisation of the first term reads:

\[
\begin{align*}
S_{h,i}(k) &= S_{h,i}^{min}(k) x_{b,h}(k) + y_{b,h}(k), \\
y_{b,h}(k) &= (S_{h,i}^{max} - S_{h,i}^{min}) x_{b,h}(k), \quad y_{b,h}(k) \leq 0,
\end{align*}
\]

where \( y_{b,h}(k) \) is a new variable.

Notice that the linearisation has been achieved at the cost of adding two more variables and five constraints. The variable \( S_{h,i}(k) \) replaces \( x_{b,h}(k) S_{h}(k) \) in (27) and the constraints (42) are added to the constraints (24)-(27). Proceeding similarly with the other two terms, the exactly linearised mixed integer model of the 8th blower switching dynamics can be obtained. Due to space limitations, the overall linearised dynamics will not be written down.

### 5.4.4. Blower station model

The nonlinearity of the constraint (29) is due to the nonlinear function \( f_{b,h}(\cdot) \) that is defined by (8). Consideration of (8) shows that there are two additive and nonlinear terms in the definition of \( f_{b,h}(\cdot) \) and both of them are in the form of a product of a binary variable and another variable. Hence, these terms can be exactly linearised following the same approach as applied to the switching dynamics model. The formal presentation of the resulting relationships is omitted, as it does not bring anything novel.

### 5.4.5. Hybrid linear model predictive controller

The HLMPC optimisation problem is based on mixed integer linearisation of the performance function and constraints as described above. Otherwise it operates in exactly the same manner as the HNMPC. As only the valve
models are approximated with certain accuracy, the control actions generated by the two controllers are close to each other if this accuracy is high. Clearly, this needs to be traded against the increased dimension of the HLMPC as already stated. Nevertheless, as there are no solvers available that are capable of solving the HNMPC optimisation tasks online, the HLMPC is a realistic alternative.

6. Case study simulation results

This section presents results of application of the derived controller to the case study WWTP in Kartuzy, northern Poland. The plant diagram is illustrated in Fig. 15.

The advanced biological treatment with nutrient removal is accomplished in the activated sludge reactor designed and operated according the University of Cape Town (UCT) process. The first zone where the phosphorus is released is anaerobic. The second zone where the denitrification process is conducted is anoxic. The internal recirculation 2 of mixed liquor originates from the anoxic zone. The returned activated sludge from the bottom of the clarifiers and the internal recirculation 1 from the end of the aerobic zone (containing nitrates) are directed to the anoxic zone. The last part of the reactor (aerobic) is aerated by a diffused aeration system. This zone is divided into four compartments of various intensity of aeration. The biologically treated wastewater and biomass (activated sludge) are separated in two parallel horizontal (rectangular) secondary clarifiers. In order to ensure a high level of phosphorus removal, iron sulphate (PIX) is added to the aerobic zone to precipitate most of the remaining soluble phosphorus (simultaneous precipitation). There is also the opportunity to precipitate phosphorus in the grit chamber (pre-precipitation). The excess biological sludge is stored in a thickener, then dewatered in two centrifuges and finally chemically stabilised in lime. It is expected that the sludge will be disposed of for agricultural applications. The aeration part of the biological reactor consists of four aeration tanks of the following volumes: tank 1—700 m³, tank 2—1760 m³, tank 3—860 m³ and tank 4—1150 m³. Two blowers supply the overall aeration system. The first blower can run with two fixed speeds while the second one with variable speed, thus being controlled by an inverter.

After a blower motor has been shut down it cannot be started again within 10 min. The four throttling valves are strongly nonlinear.

The operation of the control system was simulated over 46 h under the influent scenarios shown in Figs. 16 and 17. Large variations of the wastewater inflow to the plant and its pollution can be observed. The biological processes were
modelled by applying ASM2d (Henze et al., 1999) and the overall model was calibrated based on the plant data records. The model was then implemented in a simulation package SIMBA (Simba, 2005) in order to get information data from the controlled plant. Several sampling intervals were investigated and the following ones were finally selected: \( T_u = T_l = 5 \text{ min} \). Hence, the blower standstill period was \( N_s = 2 \). The prediction horizons \( H^I_p \), \( H^U_p \) at the upper and lower levels were 10 steps each; \( H^I_p \) needs to be long enough in order to accommodate the blower holding time.

The calculations were carried out on standard PC with Pentium IV 2.6 GHz processor in Matlab and Gams environment. The calculation time at each sampling instant needed to produce the control actions to be applied to the aeration system was less than 10 s. The HLMPC optimisation task at the lower control level was solved by using the CPLEX solver, designed to solve large MIL programming problems (Gams, 2005). The SQP solver was applied to solve the NMPC optimisation task at the upper control level. The respirations in tanks 1 and 2 are illustrated in Figs. 18 and 19. It can be seen that the respiration disturbance is time-varying. Moreover, \( R_r \) varies significantly more slowly than \( S_0 \), which confirms the key assumption upon which the ULC has been derived.

The results of the DO tracking in tanks 1 and 2 are shown in Figs. 20 and 21, respectively. For tank 1, the corresponding demanded airflow and that delivered by the LLC is illustrated in Fig. 22.

In control practice typically at present, including the Kartuzy WWTP, the collector pressure \( p_c \) is kept at a constant level sufficiently high to guarantee that the required airflows can be supplied to the aeration tanks under a whole range of the operational conditions. Hence, often the valves must throttle the airflow provided by the blowers. Clearly, it is not a viable solution from the economical point of view. The \( p_c \) trajectory produced by the proposed controller is shown in Fig. 23. Far smaller
Fig. 22. Comparison of the airflows.

Fig. 23. Optimal pressure at the collector node.

Fig. 24. Speed of the variable-speed blower.

Fig. 25. The fixed-speed blower schedule.

Fig. 26. Expanded parts of the schedule.

Fig. 27. Opening angle of the valve 1.
and much less variable collector pressure values were achieved than ever seen at the plant so far.

The variable-speed blower schedule is shown in Fig. 24 while Figs. 25 and 26 illustrate the fixed-speed blower control activity. The wide speed range 1200–2900 r.p.m. of the variable-speed blower helps to follow the airflow rate trajectory determined by UCL with good accuracy. It can be seen from Fig. 26 that the standstill period is indeed obeyed by the LLC.

Finally, the opening of valve 1 is shown in Fig. 27. The throttling valve opening allowed varies from 0° to 90°. Valve 2 remains fully open all the time while valve 1 also remains wide open. This confirms that the hierarchical controller optimises the operating cost and does not throttle the airflow provided by the blowers more than that is necessary.

7. Conclusions

The paper has considered a DO reference trajectory tracking for activated sludge processes. Both nutrient and phosphorous removal from wastewater by biological treatment using an activated sludge technology have been considered. A two-level model predictive controller has been proposed and investigated. In contrast to previous work, the control of the aeration system has also been included in the overall controller design. This integrated approach has allowed catering for the essential operational constraints of the aeration system in a proactive manner. This, in turn, has led to the least energy cost and robust DO tracking. The properties and tracking performance of the controller have been investigated by a simulation based on the data records from Kartuzy WWTP and very promising results have been obtained. The computing time needed by the controller to generate the control actions is small and its real-time implementation is very feasible.

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